Emissivity and Temperature of the Yankee Environmental Systems Hotplate

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Introduction

The Yankee Environmental Systems (YES) Hotplate is used to measure precipitation rate. This is accomplished by measuring the electrical power needed to keep the hotplate at a prescribed temperature. An equation describing the energy budget of the Hotplate is:

$$\dot{Q}_{sen} = \dot{Q}_{der} - \varepsilon_{hp} \cdot \sigma \cdot T_{hp}^{4} + \varepsilon_{hp} \cdot \varepsilon \cdot \sigma \cdot T^{4} \qquad (1)$$

Here the four terms are the electrical power supplied to the hotplate ($\dot{Q}_{sen}$), sensible heat output by the hotplate ($\dot{Q}_{der}$), radiative heat output by the hotplate ($\varepsilon_{hp} \cdot \sigma \cdot T_{hp}^{4}$) and the radiative heat input to the hotplate ($\varepsilon_{hp} \cdot \varepsilon \cdot \sigma \cdot T^{4}$). Constants in Equation 1 are the infrared emissivity of the hotplate (ε_{hp}), the Stephan-Boltzmann constant (σ), the temperature of the hotplate (T_{hp}), the infrared emissivity of the hotplate's surroundings (ε) and the ambient temperature (T). If it is assumed that the hotplate is not ventilated, then Equation 1 simplifies to Equation 2 (Borkhuu, 2009).

$$\dot{Q}_{sen} = A_2 \cdot L \cdot k(\overline{T}) \cdot (T_{hp} - T) - \varepsilon_{hp} \cdot \sigma \cdot T_{hp}^{4} + \varepsilon_{hp} \cdot \varepsilon \cdot \sigma \cdot T^{4} \qquad (2)$$

Symbols introduced in Equation 2 are a hotplate calibration constant (Borkhuu, 2009; A_2 =59.0, dimensionless), the diameter of the hotplate (L =0.13 m), and the thermal conductivity of air (k, J m^{-1} s^{-1} K^{-1}) evaluated at the average temperature ($\overline{T} = (T_{hp} + T)/2$).

Measurements

The purpose of this study is to determine values of the hotplate temperature (T_{hp}) and emissivity (ε_{hp}). The values of T_{hp} and ε_{hp} were derived by following a three step procedure. First, measurements of $\dot{Q}_{sen}$ were made in two environments with different temperatures. The two environments were in the lab (1), and in a container (2). The temperatures of the environments were T_1 = 293.45 K, and T_2 = 306.85 K, with $\dot{Q}_{sen,1}$ =10.79 J/s and $\dot{Q}_{sen,2}$ =7.94 J/s.

Second, the measurements of $\dot{Q}_{sen}$ and T were used to define two states of the hotplate:

$$\dot{Q}_{sen,1} = A_2 \cdot L \cdot k(\overline{T}_1)\cdot(T_{hp} - T_1) - \varepsilon_{hp} \cdot \sigma \cdot T_{hp}^4 + \varepsilon_{hp} \cdot \varepsilon \cdot \sigma \cdot T_1^4 \tag{3}$$

$$\dot{Q}_{sen,2} = A_2 \cdot L \cdot k(\overline{T}_2)\cdot(T_{hp} - T_2) - \varepsilon_{hp} \cdot \sigma \cdot T_{hp}^4 + \varepsilon_{hp} \cdot \varepsilon \cdot \sigma \cdot T_2^4 \tag{4}$$

Here the subscripts “1” and “2” refer to the two environments, laboratory and container, respectively.

In the third step, the IDL procedure known as “Newton” was used to evaluate T_{hp} and ε_{hp}. A copy of the IDL program is provided in the Appendix. In those calculations the emissivity of the hotplate’s surroundings was assumed equal to ε =1.0. This estimate is reasonable for the ceiling tiles of the laboratory (plaster panels), but may overestimate the emissivity of the ceiling of the container (anodized aluminum, http://www.engineeringtoolbox.com/emissivity-coefficients-d_447.html). Future work is needed to examine the limitation of this assumption.

Results

Using the IDL procedure “Newton”, values of T_{hp} and ε_{hp} were derived. Results are T_{hp} =341.70 K (68.55 ^{o}C) and ε_{hp} =0.11. The former is colder than the temperature published in Rasmussen et al. (2010), by about 6 ^{o}C, but there may be a good physical explanation for this disparity. The published temperature corresponds to what is measured at a location layered between the hotplate’s electrical resistance heater and the lower (internal) surface of the hotplate. The temperature we derived is the temperature at the plate/air boundary.

The latter temperature could be smaller than the internal temperature because of a gradient between the heated interior and the plate/air boundary. The derived emissivity is consistent with the value reported for "sheet" aluminum (http://www.engineeringtoolbox.com/emissivity-coefficients-d_447.html).

References

Borkhuu, B., Snowfall at a High Elevation Site: Comparisons of Six Measurement Techniques, M.S. Thesis, Department of Atmospheric Science, University of Wyoming, 2009

Rasmussen, R., Hallett, J., Purcell, R., Landolt, S.D., Cole, J., The hotplate snow gauge, Submitted to Journal of Applied Meteorology and Climatology, 2010

Appendix

(The appendix is IDL programing which impliments "NEWTON" and calculates T_{hp} and ε_{hp}.

The program is provided on the following page).

```
;
pro alpha_betav8
;
defsysv, '!tkmelt', 273.15d
defsysv, '!q 1', 10.79d
defsysv, '!q 3', 7.94d
defsysv, '!tk 1', 20.3d + !tkmelt
defsysv, '!tk 3', 33.7d + !tkmelt
defsysv, '!char length', 0.13d
defsysv, '!area', 0.018
defsysv, '!sigma', 5.67d-8
;
defsysv, '!ceiling emissivity', 1.0d
defsysv, '!bugee_2', 59.d
;
result = newton([60.d + !tkmelt, 1.d], 'func', /double)
alpha = result[0]
beta  = result[1]
;
print, 'Here is the assumed emmisivity for the surroundings', !ceiling_emissivity
print, 'Here is alpha = ', alpha
print, 'Here is Beta = ', beta
print, 'Here is the plate temperature = ', alpha - !tkmelt
print, 'Here is the plate emissivity = ', beta / !area
;
tk ave 1 = (alpha + !tk_1) / 2.d
;tk ave 1 = !tk 1
print,'Here is sensible heating term, environment 1, in Watt = ', !bugee 2*!char length*cond air(tk ave 1)*(alpha - !tk 1), format='(a,f
print,'Here is the output radiative term in Watt = ', beta*!sigma*alpha^4, format='(a,f10.3)'
print,'Here is the estimated hotplate emissivity = ', beta/!area, format='(a,f10.3)'
print,'Here is the input radiative term in Watt = ', beta*!ceiling emissivity*!sigma*!tk 1^4, format='(a,f10.3)'
print,'Here is residual 1 = ', !q 1 - !bugee 2*!char length*cond air(tk ave 1)*(alpha - !tk 1) - beta*!sigma*alpha^4 + beta*!ceiling emi
;
tk ave 3 = (alpha + !tk_3) / 2.d
;tk ave 3 = !tk 3
print,'Here is sensible heating term, environment 3, in Watt = ', !bugee 2*!char length*cond air(tk ave 3)*(alpha - !tk 3), format='(a,f
print,'Here is the output radiative term in Watt = ', beta*!sigma*alpha^4, format='(a,f10.3)'
print,'Here is the estimated hotplate emissivity = ', beta/!area, format='(a,f10.3)'
print,'Here is the input radiative term in Watt = ', beta*!ceiling emissivity*!sigma*!tk 3^4, format='(a,f10.3)'
print,'Here is residual 3 = ', !q 3 - !bugee 2*!char length*cond air(tk ave 3)*(alpha - !tk 3) - beta*!sigma*alpha^4 + beta*!ceiling emi
end
;
;
function func, x
 print, x
 tk ave 1 = (x[0] + !tk 1) / 2.d
 tk ave 3 = (x[0] + !tk_3) / 2.d
 ;tk ave 1 = !tk 1
 ;tk ave 3 = !tk 3
 return, [!q 1 - !bugee 2*!char length*cond air(tk ave 1)*(x[0] - !tk 1) - x[1]*!sigma*x[0]^4 + x[1]*!ceiling emissivity*!sigma*!tk 1^4,
          !q_3 - !bugee_2*!char_length*cond_air(tk_ave_3)*(x[0] - !tk_3) - x[1]*!sigma*x[0]^4 + x[1]*!ceiling_emissivity*!sigma*!tk_3^4]
end
;
;..see thermal_conductivity_quadradic.pro, the SMT Table 113 values are fitted to a quadradic
;
function cond air, tk
a = [-0.000647566472339d, 0.000110702592803d, -0.000000071116018d]
return, a[0] + a[1]*tk + a[2]*tk*tk
end
```