Emissivity and Temperature of the Yankee Environmental Systems Hotplate

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Introduction

The Yankee Environmental Systems (YES) Hotplate is used to measure precipitation rate. This is accomplished by measuring the electrical power needed to keep the hotplate at a prescribed temperature. An equation describing the energy budget of the Hotplate is:

\[ \dot{Q}_{\text{sen}} = \dot{Q}_{\text{der}} - \varepsilon_{hp} \cdot \sigma \cdot T_{hp}^4 + \varepsilon_{hp} \cdot \varepsilon \cdot \sigma \cdot T^4 \]  (1)

Here the four terms are the electrical power supplied to the hotplate (\( \dot{Q}_{\text{sen}} \)), sensible heat output by the hotplate (\( \dot{Q}_{\text{der}} \)), radiative heat output by the hotplate (\( \varepsilon_{hp} \cdot \sigma \cdot T_{hp}^4 \)) and the radiative heat input to the hotplate (\( \varepsilon_{hp} \cdot \varepsilon \cdot \sigma \cdot T^4 \)). Constants in Equation 1 are the infrared emissivity of the hotplate (\( \varepsilon_{hp} \)), the Stephan-Boltzmann constant (\( \sigma \)), the temperature of the hotplate (\( T_{hp} \)), the infrared emissivity of the hotplate’s surroundings (\( \varepsilon \)) and the ambient temperature (\( T \)). If it is assumed that the hotplate is not ventilated, then Equation 1 simplifies to Equation 2 (Borkhuu, 2009).

\[ \dot{Q}_{\text{sen}} = A_2 \cdot L \cdot k(T) \cdot (T_{hp} - T) - \varepsilon_{hp} \cdot \sigma \cdot T_{hp}^4 + \varepsilon_{hp} \cdot \varepsilon \cdot \sigma \cdot T^4 \]  (2)

Symbols introduced in Equation 2 are a hotplate calibration constant (Borkhuu, 2009; \( A_2 = 59.0 \), dimensionless), the diameter of the hotplate (\( L = 0.13 \) m), and the thermal conductivity of air (\( k \), J m\(^{-1}\) s\(^{-1}\) K\(^{-1}\)) evaluated at the average temperature (\( T = (T_{hp} + T) / 2 \)).
Measurements

The purpose of this study is to determine values of the hotplate temperature ($T_{hp}$) and emissivity ($\varepsilon_{hp}$). The values of $T_{hp}$ and $\varepsilon_{hp}$ were derived by following a three step procedure. First, measurements of $\dot{Q}_{sen}$ were made in two environments with different temperatures. The two environments were in the lab (1), and in a container (2). The temperatures of the environments were $T_1 = 293.45$ K, and $T_2 = 306.85$ K, with $\dot{Q}_{sen,1} = 10.79$ J/s and $\dot{Q}_{sen,2} = 7.94$ J/s.

Second, the measurements of $\dot{Q}_{sen}$ and $T$ were used to define two states of the hotplate:

$$\dot{Q}_{sen,1} = A_2 \cdot L \cdot k(\bar{T}_1) \cdot (T_{hp} - T_1) - \varepsilon_{hp} \cdot \sigma \cdot T_{hp}^4 + \varepsilon_{hp} \cdot \varepsilon \cdot \sigma \cdot T_1^4 \quad (3)$$

$$\dot{Q}_{sen,2} = A_2 \cdot L \cdot k(\bar{T}_2) \cdot (T_{hp} - T_2) - \varepsilon_{hp} \cdot \sigma \cdot T_{hp}^4 + \varepsilon_{hp} \cdot \varepsilon \cdot \sigma \cdot T_2^4 \quad (4)$$

Here the subscripts “1” and “2” refer to the two environments, laboratory and container, respectively.

In the third step, the IDL procedure known as “Newton” was used to evaluate $T_{hp}$ and $\varepsilon_{hp}$. A copy of the IDL program is provided in the Appendix. In those calculations the emissivity of the hotplate’s surroundings was assumed equal to $\varepsilon = 1.0$. This estimate is reasonable for the ceiling tiles of the laboratory (plaster panels), but may overestimate the emissivity of the ceiling of the container (anodized aluminum, http://www.engineeringtoolbox.com/emissivity-coefficients-d_447.html). Future work is needed to examine the limitation of this assumption.

Results

Using the IDL procedure “Newton”, values of $T_{hp}$ and $\varepsilon_{hp}$ were derived.

Results are $T_{hp} = 341.70$ K (68.55 °C) and $\varepsilon_{hp} = 0.11$. The former is colder than the temperature published in Rasmussen et al. (2010), by about 6 °C, but there may be a good physical explanation for this disparity. The published temperature corresponds to what is measured at a location layered between the hotplate’s electrical resistance heater and the lower (internal) surface of the hotplate. The temperature we derived is the temperature at the plate/air boundary.
The latter temperature could be smaller than the internal temperature because of a gradient between the heated interior and the plate/air boundary. The derived emissivity is consistent with the value reported for “sheet” aluminum (http://www.engineeringtoolbox.com/emissivity-coefficients-d_447.html).

References

Borkhuu, B., Snowfall at a High Elevation Site: Comparisons of Six Measurement Techniques, M.S. Thesis, Department of Atmospheric Science, University of Wyoming, 2009


Appendix

(The appendix is IDL programing which impliments “NEWTON” and calculates $T_{hp}$ and $\varepsilon_{hp}$. The program is provided on the following page).
defsysv, '!tkmelt', 273.15d
defsysv, '!q_1', 10.79d
defsysv, '!q_3', 7.94d
defsysv, '!tk_1', 20.3d + !tkmelt
defsysv, '!tk_3', 33.7d + !tkmelt
defsysv, '!char_length', 0.13d
defsysv, '!area', 0.018
defsysv, '!sigma', 5.67d-8
; defsysv, '!ceiling_emissivity', 1.0d
defsysv, '!bugee_2', 59.0d
result = newton([60.d + !tkmelt, 1.d], 'func', /double)
alpha = result[0]
beta  = result[1]

print, 'Here is the assumed emissivity for the surroundings', !ceiling_emissivity
print, 'Here is alpha = ', alpha
print, 'Here is Beta = ', beta
print, 'Here is the plate temperature = ', alpha - !tkmelt
print, 'Here is the plate emissivity = ', beta / !area

; tk_ave_1 = (alpha + !tk_1) / 2.d
; tk_ave_1 = !tk_1
print, 'Here is sensible heating term, environment 1, in Watt = ', !bugee_2!*char_length*cond air(tk_ave_1)*(alpha - !tk_1), format='(a,f)
print, 'Here is the output radiative term in Watt = ', beta*!sigma*alpha^4, format='(a,f10.3)
print, 'Here is the input radiative term in Watt = ', beta*!sigma*alpha^4 + beta*!ceiling emissivity*!sigma*!tk_1^4, format='(a,f10.3)
print, 'Here is residual 1 = ', !q_1 - !bugee_2!*char_length*cond air(tk_ave_1)*(alpha - !tk_1) - beta*!sigma*alpha^4 + beta*!ceiling emissivity*!sigma*!tk_1^4

; tk_ave_3 = (alpha + !tk_3) / 2.d
; tk_ave_3 = !tk_3
print, 'Here is sensible heating term, environment 3, in Watt = ', !bugee_2!*char_length*cond air(tk_ave_3)*(alpha - !tk_3), format='(a,f)
print, 'Here is the output radiative term in Watt = ', beta*!sigma*alpha^4, format='(a,f10.3)
print, 'Here is the estimated hotplate emissivity = ', beta / !area, format='(a,f10.3)
print, 'Here is the input radiative term in Watt = ', beta*!ceiling_emissivity*!sigma*!tk_3^4, format='(a,f10.3)
print, 'Here is residual 3 = ', !q_3 - !bugee_2!*char_length*cond air(tk_ave_3)*(alpha - !tk_3) - beta*!sigma*alpha^4 + beta*!ceiling emissivity*!sigma*!tk_3^4
end

; function func, x
print, x
tk_ave_1 = (x[0] + !tk_1) / 2.d
tk_ave_3 = (x[0] + !tk_3) / 2.d
;tk_ave_1 = !tk_1
;tk_ave_3 = !tk_3
return, [!q_1 - !bugee_2!*char_length*cond air(tk_ave_1)*(x[0] - !tk_1) - x[1]!sigma*x[0]^4 + x[1]!ceiling emissivity*!sigma*!tk_1^4,
!q_3 - !bugee_2!*char_length*cond air(tk_ave_3)*(x[0] - !tk_3) - x[1]!sigma*x[0]^4 + x[1]!ceiling emissivity*!sigma*!tk_3^4]
end
;..see thermal_conductivity_quadradic.pro, the SMT Table 113 values are fitted to a quadratic

; function cond air, tk
a = [-0.000647566472339d, 0.000110702592803d, -0.000000071116018d]
return, a[0] + a[1]*tk + a[2]*tk*tk