**Review of oceanic dynamics:**

Horizontal equations of motion (velocity component form):

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv + A_z \frac{\partial^2 u}{\partial z^2}$$

$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu + A_z \frac{\partial^2 v}{\partial z^2}$$

Terms:

Acceleration

Horizontal pressure gradient term (the pressure gradient force per unit mass)

Coriolis term (the Coriolis force per unit mass)

Friction term (the friction force per unit mass)

***Inertial Flow*:**

Assessment of inertial flow requires an examination of the horizontal equations of motion in the situation where the acceleration and the Coriolis terms dominate. In that situation, here are the x- and y-component equations of motion:

$$\frac{du}{dt} = fv$$

$$\frac{dv}{dt} = -fu$$

Rearranging:

$$0 = fv - \frac{du}{dt}$$

$$0 = -fu - \frac{dv}{dt}$$

Derivation of a flow consistent with these two coupled differential equations requires calculus.

Note: This derivation will contrast with the *geostrophic* solution. In the case of geostrophy, we obtained an expression for the velocity by rearranging the equation of motion (i.e., no calculus was required to derive Equation 2).

The solution of the coupled x- and y- component equations, both ordinary differential equations (ODEs), requires complex algebra and calculus:

$$i \cdot \left\{ 0 = fv - \frac{du}{dt} \right\}$$

$$0 = -fu - \frac{dv}{dt}$$

Note: $i$ is defined as the square root of negative one, that is, $i = \sqrt{-1}$

Rearranging:

$$0 = fvi - \frac{d(ui)}{dt}$$

$$0 = -fu - \frac{dv}{dt}$$

Adding the x and y component equations:

$$0 = f(vi - u) - \frac{d(v + ui)}{dt}$$

Multiply the first term by $i^2$ and the second by -1

$$0 = if(vi^2 - ui) + \frac{d(v + ui)}{dt}$$

This is what we get:

$$0 = -if(v + ui) + \frac{d(v + ui)}{dt} \tag{3}$$

Solutions to (3) have the following form:

$$v + ui = h_o \, exp(ift) \qquad (4)$$

In Equation 4 we have a yet-to-be-determined parameter ($h_o$).

Relevant to Equation 4 is Euler's theorem.

$$\exp(i \cdot \theta) = \cos(\theta) + i \cdot \sin(\theta) \qquad (5)$$

The x and y component velocities are obtained from (3), (4) and (5). Note: the velocity components oscillate with time

$$u(t) = h_o \, sin(ft)$$

$$v(t) = h_o \, cos(ft)$$

The velocity vector (the flow) is then

$$\vec{V} = h_o \, sin( \, ft \,)\hat{i} + h_o \, cos( \, ft \,)\hat{j}$$

and the magnitude of the flow (the speed) is

$$V^2 = u(t)^2 + v(t)^2 = h_o^2 \, sin( \, ft \,)^2 + h_o^2 \, cos( \, ft \,)^2 = h_o^2 \left( sin( \, ft \,)^2 + cos( \, ft \,)^2 \right) = h_o^2$$

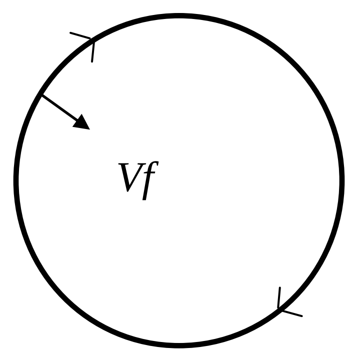
Hence, the parameter, $h_o$ , is the speed of the flow:

$$u(t) = V \, sin( \, ft \,)$$

$$v(t) = V \, cos( \, ft \,)$$

Also, the speed ($V$) can be expressed in terms of $f$ and the radius of the trajectory ($R$):

Assume Northern hemisphere
Acceleration=Coriolis
$V^2 / R = Vf$
$V = Rf$



Some things to note about Inertial flow:

1. The period of the oscillation depends on latitude,

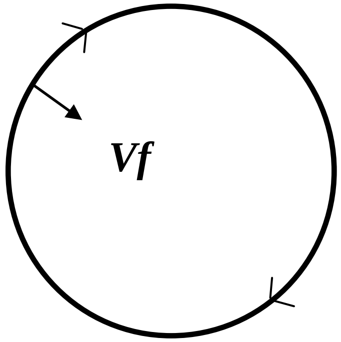
$$\textit{period of oscillation} = \frac{2\pi}{f} = \frac{\pi}{\Omega \sin(\varphi)}$$

2. The flow trajectory is a circle in u/v space, and the speed is proportional to the radius of the inertial circle

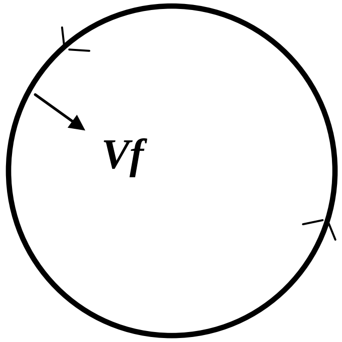
$$V = \sqrt{u^2 + v^2} = Rf$$

3. The circulation is clockwise in the NH and counterclockwise in the SH, consistent with the forcing provided by the Coriolis force (to the right of the flow in NH and to the left of the flow in the SH)

Northern hemisphere (NH)




Southern hemisphere (SH)



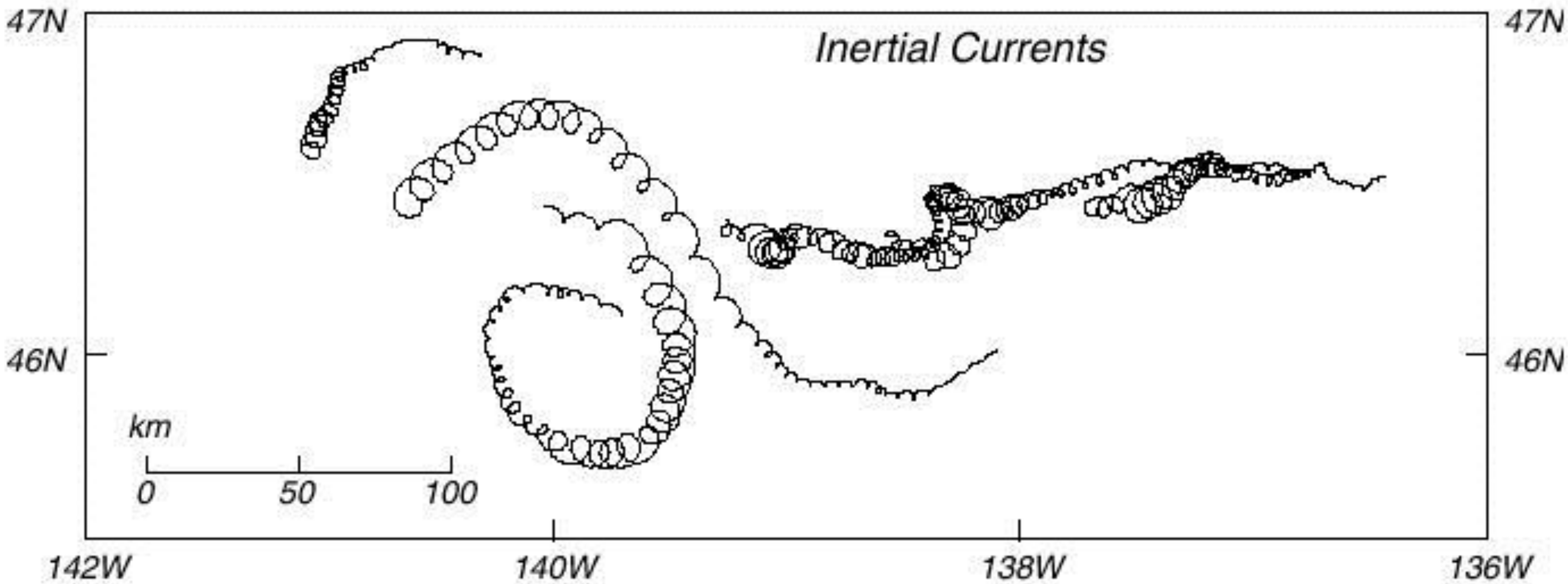



Figure 9.1 Inertial currents in the North Pacific in October 1987 (days 275–300) measured by holey-sock drifting buoys drogued at a depth of 15 meters. Positions were observed 10–12 times per day by the Argos system on NOAA polar-orbiting weather satellites and interpolated to positions every three hours. The largest currents were generated by a storm on day 277. Note these are not individual eddies. The entire surface is rotating. A drogue placed anywhere in the region would have the same circular motion. From van Meurs (1998).