1) Differentiate the following functions (write f'(x)):

a) f(x) = x

b) f(x) = x^2

c) f(x) = ln(x)

d) f(x) = exp(x)

e) f(x) = sin(x)

f) f(x) = 1/x

2) Integrate the following functions (write $\int f(x)dx$):

a) f(x) = x

b) f(x) = x^2

c) f(x) = ln(x)

d) f(x) = exp(x)

e) f(x) = sin(x)

f) f(x) = 1/x

3) A function whose derivative is a constant multiple of itself must be?

a) periodic

b) linear

c) exponential

d) quadratic

e) logarithmic

4) Explain your answer to problem #3 by examining your answers to problem #1. For example, the derivative of f(x) = x^2 is 2x, and that is not a constant multiplied by the function f(x). Hence, f(x) = x^2 is not a correct answer to problem #3. Hence, a quadratic function is not a correct answer to problem #3.

Power Function:

$$\frac{d\left(x^n\right)}{dx} = n \cdot x^{n-1}$$

Exponential function:

$$\frac{d\left(\exp(x)\right)}{dx} = \exp(x)$$

Periodic Function:

$$\frac{d\left(\sin(x)\right)}{dx} = \cos(x)$$

$$\frac{d\left(\cos(x)\right)}{dx} = -\sin(x)$$

Logarithm:

$$\frac{d\left(\ln(x)\right)}{dx} = \frac{1}{x}$$

Integration requires a recognition of differentials. Almost all of the problems we work deal with are *definite* integrals:

$$\int_A^B f(x)dx = [F(x)]_A^B$$

In the previous equation, we know the form of the function $f(x)$ and we are searching for the function $F(x)$. The function $F(x)$ – the antiderivative – has the property that its derivative is equal to the integrand.

In the previous equation, the "$[\ ]$" notation is shorthand notation for this: evaluate the antiderivative at its upper limit (B) and subtract from that the antiderivative evaluated at the lower limit (A).

$$\int_A^B x dx = \left[\frac{1}{2}\cdot x^2\right]_A^B = \frac{1}{2}\cdot\left(B^2 - A^2\right)$$

$$\int_A^B x^2 dx = \left[\frac{1}{3}\cdot x^3\right]_A^B = \frac{1}{3}\cdot\left(B^3 - A^3\right)$$

$$\int_A^B \exp(x)dx = [\exp(x)]_A^B = \exp(B) - \exp(A)$$

$$\int_A^B \sin(x)dx = [-\cos(x)]_A^B = -(\cos(B) - \cos(A))$$

$$\int_A^B \frac{1}{x} dx = [\ln(x)]_A^B = \ln(B) - \ln(A)$$

We know this (a function whose derivative is a constant multiple of itself):

$$f' = k \cdot f$$

Formally

$$\frac{df}{dx} = k \cdot f$$

Rearranging….because we are going to integrate and thus derive the form of f

$$\frac{1}{f} \cdot df = k \cdot dx$$

Integrating both sides

$$\int_{f_o}^{f} \frac{1}{f} \cdot df = k \int_{x_o}^{x} (1) \cdot dx$$

Antiderivatives

$$\left[\ln(f)\right]_{f_o}^{f} = k\left[x\right]_{x_o}^{x}$$

Algebra

$$\ln(f) - \ln(f_o) = k\left(x - x_o\right)$$

$$\ln\left(\frac{f}{f_o}\right) = k\left(x - x_o\right)$$

$$\frac{f}{f_o} = \exp\left(k\left(x - x_o\right)\right)$$

$$f(x) = f_o \cdot \exp\left(k\left(x - x_o\right)\right)$$

Note: $f' = k \cdot f$