Threshold Radar Reflectivity for Drizzling Clouds

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Abstract

Empirical studies have suggested the existence of a threshold radar reflectivity between nonprecipitating and precipitating clouds; however, there has been neither a rigorous theoretical basis for the threshold reflectivity nor a sound explanation as to why empirically determined threshold reflectivities differ among studies. Here we present a theory for the threshold reflectivity by relating it to the autoconversion process. This theory not only demonstrates the sharp transition from cloud to rain when the radar reflectivity exceeds some value (threshold reflectivity) but also reveals that the threshold reflectivity is an increasing function of the cloud droplet concentration. The dependence of threshold reflectivity on droplet concentration suggests that the differences in empirically determined threshold reflectivity arise from the differences in droplet concentration. The favorable agreement with measurements collected over a wide range of conditions further provides observational support for the theoretical formulation. The results have many potential applications, esp. to remote sensing of cloud properties and the second aerosol indirect effect.

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1. Introduction

Low-level clouds occur over large areas, especially in the eastern parts of the oceans with generally cooler water surface temperatures. These boundary-layer clouds play a crucial role in shaping the Earth’s climate, and calls for long-term and global observations of these clouds, which further demands remote sensing techniques.

Millimeter-wave cloud radars have found increasing application in remote sensing of cloud properties [Clothiaux et al. 1995; Moran et al. 1998; Galloway et al. 1999; Stephens et al. 2002; Kollias et al. 2005]. Cloud radars operating at millimeter wavelengths generally receive greater echo intensity from cloud droplets than conventional precipitation radars operating at longer wavelengths (e.g., centimeters) because radar backscattering of spherical water droplets/drops decreases with increasing radar wavelength. In drizzle-free clouds, the radar reflectivity is a good measure of the cloud liquid water content [Wang and Geerts 2003]. However, in precipitating clouds, the radar reflectivity may be dominated by the presence of few drizzle-sized drops [Fox and Illingworth 1997; Vali et al. 1998]. Thus, different algorithms are needed for retrieving cloud properties of nonprecipitating and precipitating clouds, and distinguishing between nonprecipitating and precipitating clouds is critical for using remote sensing techniques to retrieve cloud properties. Identification of drizzle occurrence is also essential for studying non-adiabatic behavior of clouds [Chin et al. 2000].

Several studies have proposed a threshold in radar reflectivity as the basis of discrimination between non-precipitating and precipitating clouds. However, there appears to be no physically sound approach for specification of the threshold reflectivity,
and a wide range of values has been assigned to it. For example, Chin et al. [2000] used the threshold of –15 dBZ as an indication of drizzle occurrence in their microphysical retrieval of continental stratiform clouds. Kato et al. [2001] used –20 dBZ in their retrieval of continental stratus. Matrosov et al. [2004] found a gradual deterioration of the liquid water content retrieved from radar reflectivity when reflectivity threshold set for the retrieval increases, and at high reflectivity values an ambiguity exists between clouds with high liquid water contents and those with drizzle presence. Kogan et al. [2005] used –17 dBZ as the reflectivity threshold to partition their observations into non-precipitating and precipitating clouds. They also examined the influence of varying the threshold between –20 and –15 dBZ on their results.

Heretofore, the issue of the threshold reflectivity for precipitation presence has been examined primarily by empirical analysis of observational data, and a quantitative theoretical investigation is lacking. The cause of the difference between various empirically determined threshold reflectivities remains largely unknown. The primary objective of this contribution is to derive the threshold reflectivity from first principles, and to further relate it to physically relevant cloud properties such as the cloud droplet number concentration. The theoretical formulation is also compared to observations to provide empirical support for the theory.

2. Threshold Function

2.1. Theoretical expression

Warm rain starts with the autoconversion process whereby cloud droplets grow into embryonic drizzle drops. In a series of publications [Liu et al., 2004, 2005, 2006, 2007], we have theoretically demonstrated that the cloud-to-rain transition behavior of the autoconversion process can be described by the general threshold function given by
\[ T = \left[ \frac{\int_{r_c}^{\infty} r^\delta n(r) \, dr}{\int_{0}^{r_c} r^\delta n(r) \, dr} \right]^{\gamma} \left[ \frac{\int_{r_c}^{\infty} r^\delta n(r) \, dr}{\int_{0}^{r_c} r^\delta n(r) \, dr} \right]^{\delta + q}, \quad (1) \]

where the exponent 6 in the first square bracket measures the strength of the collection process (collision and coalescence, and the exponent \( \delta \) in the second square bracket denotes the order of the power moment of the cloud droplet size distribution \( n(r) \) in question. The critical radius \( r_c \), beyond which the collection process becomes dominant, corresponds to the kinetic potential barrier of the droplet population and is a function of the droplet concentration and liquid water content [see McGraw and Liu, 2003, 2004, and Liu et al., 2004 for the kinetic potential theory and exact definition of the critical radius].

When \( \delta = 0, 3, \) and 6, Eq. (1) describes the transition behavior of the droplet concentration, liquid water content, and radar reflectivity, respectively. For the general Weibull droplet size distribution, which has been shown to well describe the cloud droplet size distribution [Liu and Hallett, 1997; Liu and Daum, 2000],

\[ n(r) = \frac{qN}{r_0^q} r^{q-1} \exp \left[ -\left( \frac{r}{r_0} \right)^q \right], \quad (2) \]

Eq. (1) becomes

\[ T = \gamma \left( \frac{6 + q}{q} \right)^{\gamma} \left( \frac{\delta + q}{q} \right)^{\gamma} x_{c\gamma}, \quad (3a) \]

\[ x_{c\gamma} = \left( \frac{r_c}{r_0} \right)^q = \Gamma^{q/3} \left( \frac{3 + q}{q} \right) x_{c\gamma}^{q/3}, \quad (3b) \]

\[ x_c = 9.7 \times 10^{-17} N^{3/2} L^{-2}. \quad (3c) \]
where q is a parameter depending on the spectral shape of the cloud droplet size
distribution; $r_0$ is the mode radius; $x_c$ is the ratio of the critical to mean masses of the
droplet population; $\Gamma$ and $\gamma$ represent the complete and incomplete Gamma functions,
respectively. When $\delta = 0$, $T$ becomes the number threshold function, and is given by

$$T_N = \gamma\left(\frac{6+q}{q}, x_{cq}\right) \gamma(1, x_{cq})$$  \hspace{1cm} (3d)

For radar applications, it is desirable to express the threshold function, or, $x_c$, in terms of
the radar reflectivity factor $Z$ such that

$$x_c = 9.7 \times 10^{-17} 64 \left(\frac{3}{4 \pi \rho_w}\right)^2 \Gamma\left(\frac{6+q}{q}\right) \Gamma^{-2} \left(\frac{3+q}{q}\right) N^{1/2} Z^{-1}.$$  \hspace{1cm} (4a)

$$Z = 64 \int r^n n(r) dr.$$  \hspace{1cm} (4b)

Substitution of Eq. (4) into Eq. (3d) gives the equation that quantifies the dependence of
the number threshold function on the radar reflectivity as well as droplet concentration
and the spectral shape of the cloud droplet size distribution. Utilizing the typical droplet
size distribution with $q = 3$ yields a simpler number threshold function given by

$$T_N = \gamma(3, x_c) \gamma(1, x_c),$$  \hspace{1cm} (5a)

$$x_c = 7.1 \times 10^{-16} N^{1/2} Z^{-1}.$$  \hspace{1cm} (5b)

Figure 1 shows the dependence of the number threshold function on radar
reflectivity for different values of the droplet concentration: $N = 50$ cm$^{-3}$, $N = 500$ cm$^{-3}$,
and $N = 1000$ cm$^{-3}$. Two features stand out in this figure. One is that the change of the
threshold function with reflectivity is like experiencing a phase transition ---- first the
threshold function changes little when radar reflectivity increases; when the reflectivity
reaches a certain value it increases sharply, and then the threshold function remains
almost unchanged with further increasing radar reflectivity. This provides theoretical support for the common practice of using a threshold reflectivity to separate precipitating from non-precipitating clouds. The other feature is that with increasing droplet concentration, the “phase-transition point” shifts to the right, suggesting that the threshold reflectivity is not a constant as commonly assumed, but increases with increasing droplet concentration instead. This important point will be further examined in Section 3.

2.2. Comparison with observations

Wang and Geerts [2003] proposed an approach to empirically examine the transition from non-precipitating to precipitating clouds by determining the occurrence probability of drizzle-sized drops for a given radar reflectivity value. Drizzle was deemed present if the particle count of the Particle Measurement Systems 2D-C probe exceeded zero. The reflectivity was measured concurrently by the Wyoming Cloud Radar (WCR; Vali et al. 1998) onboard the same aircraft. Data from the first uncontaminated radar gate of a side-looking radar beam were used. This gate was 75-90 m displaced horizontally from the 2D-C probe, and the reflectivity data, sampled at ~30 Hz, were averaged along-track to 1 Hz to match the 2D-C data frequency. The resulting reflectivity (Z) values were binned in integer increments, i.e. the bin size is 1 dB. The probability of drizzle at a given value of Z is defined as the number of occurrences in this Z bin with drizzle presence as defined above divided by all occurrences in this bin. The probabilities were computed by accumulating occurrences in all Z bins for all flight legs during any flight. The cumulative length of these flight legs varied between 292-705 km for the six flights used in this study. Three of the flights were conducted in summer 1999 within 100 km of the
Oregon coast (Wang and Geerts 2003). The three other flights were conducted 260-670 km offshore the S. Oregon and N. California coasts, in summer 2006. All six flights were conducted during the daytime.

The mean cloud droplet number concentration (N) was derived from the FSSP probe, as an average for the same cumulative flight length, for each flight. The combination of measurements near shore and in clean marine environments allows for a large range of droplet concentrations. Applying the droplet concentrations for the six cases analyzed here to Eq. (5), we calculate the theoretical threshold function for each case. Figure 2 compares the theoretical threshold functions with the observational results determined using the Wang-Geerts approach. It is clear that the theoretical threshold function well describes the observational results in general, providing observational support for the theoretical formulation on one hand, and theoretical support for the Wang-Geerts approach on the other. According to Eq. (4), the discrepancies between theoretical predictions and observations may arise from the differences in the spectral shape of the cloud droplet size distribution. The relative broadness of the observed transition region from non-precipitating to precipitating reflectivities, compared to generally steeper theoretical curves, may also be due to the variability in droplet size distributions. Furthermore, some drizzle drops may escape detection of the 2D-C probe because of its small sampling volume, leading to underestimated drizzle probability. Drizzle drops may be also affected by other processes such as accretion, especially for cases with large reflectivities.

3. Dependence of Threshold Reflectivity on Droplet Concentration
Previous studies of threshold reflectivity have been based primarily on observational analysis, and a range of values has been suggested for the threshold reflectivity. For example, studies based on in situ measurements of cloud drop size distributions for marine [Frisch et al., 1995a, b; Baedi et al. 2002] and continental [Baedi et al. 2002] clouds have demonstrated the presence of a sharp increase in radar reflectivity associated with drizzle formation over the range from –20 to –15 dBZ, suggesting the existence of a reflectivity threshold that separates non-precipitating from precipitating clouds. By analyzing in situ microphysical data, Sauvageot and Omar [1981] found a threshold of –15 dBZ for continental stratocumulus clouds. Frisch et al. [1995a, b] indicated that radar reflectivities lower than –18 dBZ are usually associated with non-precipitating clouds whereas reflectivities greater than –16 dBZ tend to be correlated with the presence of droplets of diameter ≥ 50 µm. Mace and Sassen [2000] determined that for continental clouds observed over the ARM SGP site, layers with maximum reflectivity ≥ -20 dBZ nearly always contain drizzle. By analyzing in-situ measurements of droplet/drop spectra obtained in marine, coastal and continental clouds, Baedi et al. [2002] showed that the maximum radar reflectivity due to the non-drizzling parts of clouds is around –20 dBZ whereas the minimum reflectivity due to the drizzle component is about –10 dBZ, and on average there appears a jump of approximately 10 dBZ in reflectivity between drizzle-free and drizzle-contaminated clouds. Wang and Geerts [2003] demonstrated that in marine clouds the threshold varies between –19 and –16 dBZ for three different cases.

What causes the differences in these empirical values of threshold reflectivity? The dependence of the threshold reflectivity on droplet concentration revealed by our
theoretical results shown in Fig. 1 may provide physical insight with regard to this issue, and warrants further examination.

Although the threshold behavior is clear from Fig 1, the cloud-to-rain transition is not exactly the all-or-nothing behavior as characterized by a step function, which leads to some ambiguity in defining the threshold reflectivity. Therefore, we introduce the concept of p-threshold reflectivity, defined as the reflectivity that corresponds to the threshold function $T_N = p$. With this definition, we can derive the relationship between the p-threshold reflectivity and the droplet concentration as follows. First, according to Eq. (5a), given $T_N = p$, we can obtain a corresponding $x_{cp} = x_c(p)$. Then the p-threshold reflectivity is given by

$$Z_{cp} = 7.1 \times 10^{-16} N^{1/2} x_{cp}^{-1}. \quad (6)$$

expressing it in the unit of dBZ for $Z$, we have

$$dBZ_{cp} (mm^6 m^{-3}) = 10 \log \left(10^{12}\right) Z_{cp} \approx -31 + 5 \log N - 10 \log x_{cp} \quad (7)$$

In radar-related studies, it seems reasonable to consider $p = 0.9$, which corresponds to $x_{cp} = 0.1$. Substitution of this value into Eq. (7) yields the dependence of the 90% threshold reflectivity on the droplet concentration

$$dBZ_{cp} (mm^6 m^{-3}) = -21 + 5 \log N \quad (8)$$

Equation (8) reveals that the threshold reflectivity increases with increasing droplet concentration.

From the observational curves shown in Fig 2, we determined 5 pairs of droplet concentration and threshold reflectivity. (The 90% probability threshold reflectivity could not be determined for Case 1, which had the largest value for $N$ and very little drizzle.)
Figure 3 compares the theoretical dependence of the 90% threshold reflectivity on the droplet concentration with these observational results. It is evident from the figure that the observational results compare favorably with the theoretical expression, providing observational support for the theoretical formulation. The increase of threshold reflectivity with increasing droplet concentration is consistent with the notion that clouds with more droplets can hold more cloud water [Berg et al. 2006], and indicates that the differences in the empirical values of threshold reflectivity reported in literature likely arise from the differences in the droplet concentration between the corresponding clouds examined. Other factors being equal, an increase in droplet concentration will suppress the collection process by enhancing the critical radius and the kinetic potential barrier [McGraw and Liu, 2003, 2004, and Liu et al., 2004].

It is noteworthy that the theoretical formulation also suggests other possible reasons for the differences in empirical threshold reflectivity. For example, different researchers might have used different criteria for defining the threshold reflectivity (e.g., different p values), and clouds in question might have different spectral shapes of the cloud droplet size distribution. These issues will be addressed elsewhere.

4. Concluding Remarks

The theoretical threshold function previously derived for representing the autoconversion process in atmospheric models is related to radar reflectivity. The new formulation clearly shows a general sharp transition when radar reflectivity exceeds some threshold value, and compares favorably with observational results collected from marine stratus clouds over a wide range of conditions. A simple relationship is also derived...
between the threshold reflectivity and the droplet concentration, revealing that the threshold reflectivity increases when droplet concentration increases. This theoretical expression compares favorably with observational results as well. Such dependency of the threshold reflectivity on droplet concentration provides a physical explanation for the wide range of values that have been obtained empirically for the threshold reflectivity. The theoretical formulation also suggests other possible reasons for the differences in empirical threshold reflectivity. For example, different researchers might have used different criteria for defining the threshold reflectivity (e.g., different p values), and the clouds in question might have different spectral shapes of the cloud droplet size distribution. Furthermore, only the dependence on droplet concentration is discussed in this paper because of its close link to the Wang-Geerts approach. If the threshold function is defined with respect to other quantities such as the liquid water content, different results are expected. All these suggest the necessity to specify the criteria and the approach used in empirical determination of threshold reflectivity.

The theoretical formulation presented herein has many potential applications. For example, the positive dependence of the threshold reflectivity on the droplet concentration is consistent with the notion that clouds containing abundant cloud condensation nuclei (e.g. due to anthropogenic aerosols) can hold more cloud liquid water [Berg, 2006]. Therefore, the theoretical relationship between the threshold reflectivity can be applied to long-term cloud radar observations such as that conducted over the ARM SGP site to investigate aerosol indirect effects. A positive correlation between aerosol loading and threshold reflectivity implies plausible the 2nd aerosol indirect effect. It may also be used inversely to infer the cloud droplet concentration from
radar measurements, which of course is limited by the accuracy of the measurement of radar reflectivity.

The following two points are noted in passing. First, although the focus of this paper is radar reflectivity, the agreement between the theoretical formulation and observational results provides additional observational validation of the theoretical autoconversion parameterization we have presented previously [see Daum et al. (2007) for validation against solely in situ microphysical measurements]. Second, the effect of spectral shape of the cloud droplet size distribution on the threshold behavior and relationship between threshold reflectivity and droplet concentration is ignored at present; we plan to examine this issue in detail when additional data become available.

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References


Figure Captions

Figure 1. Dependence of the threshold function on the cloud radar reflectivity factor. The different curves represent results calculated from the analytical threshold function at different cloud droplet concentrations (N).

Figure 2. Comparison of the theoretical threshold function to the observational results. The theoretical curves are the same as in Fig. 1, but for values of N corresponding with the observed mean values of N. The colors of the theoretical curves correspond to those representing the observational results given in the figure legend. The observations were made off the coast of N. California and Oregon. The averaged droplet concentrations shown in the legend correspond to the cases (denoted by date yymmd) of 990809, 990816, 990817, 060629, 060613, 060523, respectively.

Figure 3. Dependence of reflectivity threshold on droplet concentration. The black line and the dots represent the theoretical and observational results, respectively.
Figure 1.
Figure 2.
Figure 3

90% Threshold $Z_c = -21 + 5 \log N$