

1. Ageostrophic flow interpretation (50% of the homework - 10 points each a thru e)

a. Show that, for inviscid flow and $Ro \ll 1$, the ageostrophic flow can be approximated as follows

$$(u_a, v_a) = \frac{1}{f_o} \left(-\frac{\partial v_g}{\partial t}, \frac{\partial u_g}{\partial t} \right) + \frac{1}{f_o} \left(-u_g \frac{\partial v_g}{\partial x} - v_g \frac{\partial v_g}{\partial y}, u_g \frac{\partial u_g}{\partial x} + v_g \frac{\partial u_g}{\partial y} \right)$$

AGEO isallobaric term inertial – advective term

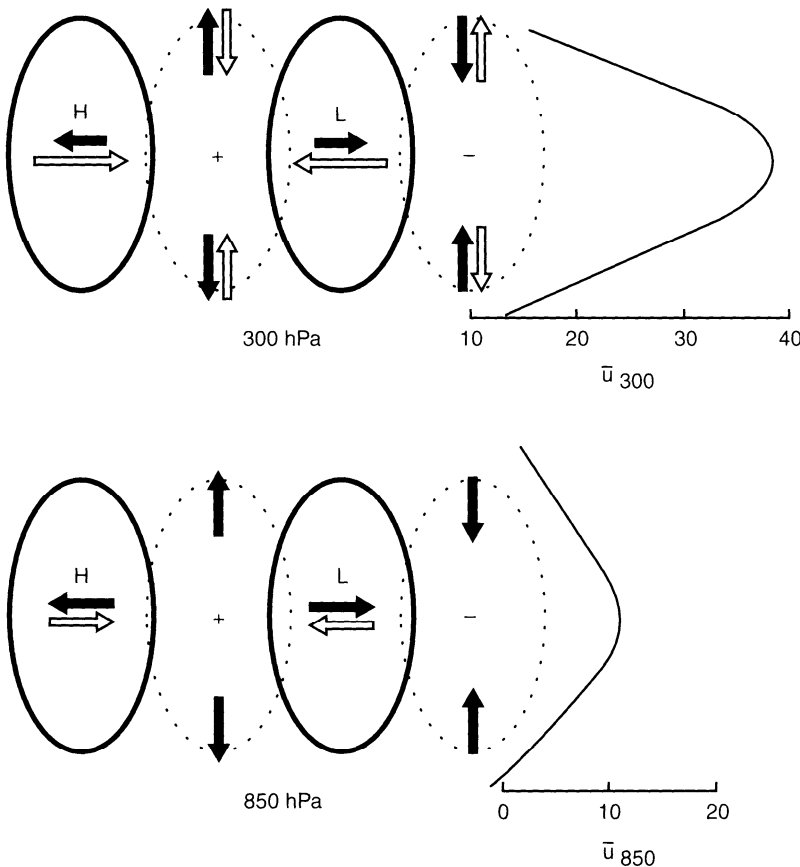
b. Show that for a zonal wavetrain ($\overline{v_g} = 0$), this can be further approximated as

$$(u_a, v_a) = -\frac{1}{f_o^2} \frac{\partial \nabla \phi}{\partial t} + \frac{1}{f_o^2} \left(-u_g \frac{\partial^2 \phi}{(\partial x)^2}, u_g \frac{\partial u_g}{\partial x} \right) \tag{1}$$

c. Note that in a broad zonal wavetrain $\zeta_g \cong \frac{1}{f_o} \frac{\partial^2 \phi}{(\partial x)^2}$ if v_g is ignored. Use equation (1) to explain the super- (sub-) geostrophic flow in ridges (trofs) in a wavetrain. Specifically, draw the wavetrain, and show relative vorticity max/min and ageostrophic flow. Also indicate, both in trofs and in ridges, what term in the equation dominates and is used to plot \vec{v}_a .

d. As (c), but now use (1) to explain the ageostrophic flow in a zonal jet streak.

e. Armed (1), explain all arrows in Fig. 6.15 in Holton (below). You may want to label the arrows or groups of arrows (A, B, C ...) to explain them



Isallobaric and advective contributions to the ageostrophic wind for baroclinic waves in westerlies. Solid ellipses indicate perturbation geopotential patterns at 300 and 850 hPa. Dashed ellipses show a geopotential tendency pattern with positive and negative tendencies indicated by + and - signs, respectively. The mean zonal flow distribution in which the waves are embedded is indicated on the right for each level. Solid arrows show the isallobaric part of the ageostrophic wind, and open arrows show the advective part. (Adapted from Lim et al., 1991.)

Fig. 6.15:

2. To demonstrate that hydrostatic balance is sufficient to explain the westward tilt of troughs with height in a baroclinic environment ($\frac{\partial \bar{T}}{\partial y} < 0$, with \bar{T} the zonal-mean temperature), assume a trough-ridge system at 1000 mb described by $\varphi = \varphi_o \sin(kx - ct)$, where φ is the geopotential (gZ), φ_o is a wave amplitude, and c is the wave speed ($c > 0$ since waves move eastward in mid-latitudes).

(a) Show that the horizontal temperature advection is quadrature phase shifted compared to φ

($-v_g \frac{\partial \bar{T}}{\partial y} = -\frac{k}{f} \frac{\partial \bar{T}}{\partial y} \varphi_o \cos(kx - ct)$), and that this results in a zonal variation of temperature as

$T = \bar{T} - T_o \sin(kx - ct)$, i.e. temperature varies in opposition with φ (trofs are warm, ridges cold).

(b) Use the thickness equation to show that, given this temperature distribution, troughs should tilt westward with height.